4-5 ResNet

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Contents

Motivation

1. Won the 1st place of ImageNet ILSVRC 2015 classification competition
2. Theoretically, the training error should go down as the number of layers increases
3. However, it is not the case

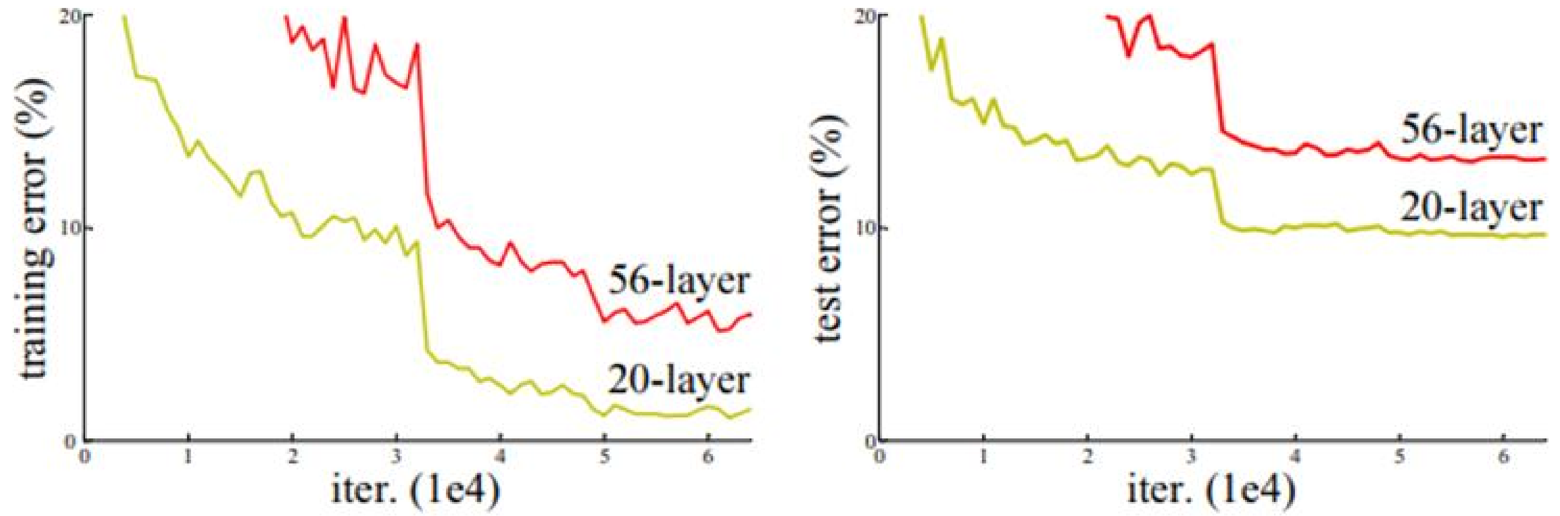


Figure 1. Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer “plain” networks. The deeper network has higher training error, and thus test error. Similar phenomena on ImageNet is presented in Fig. 4.

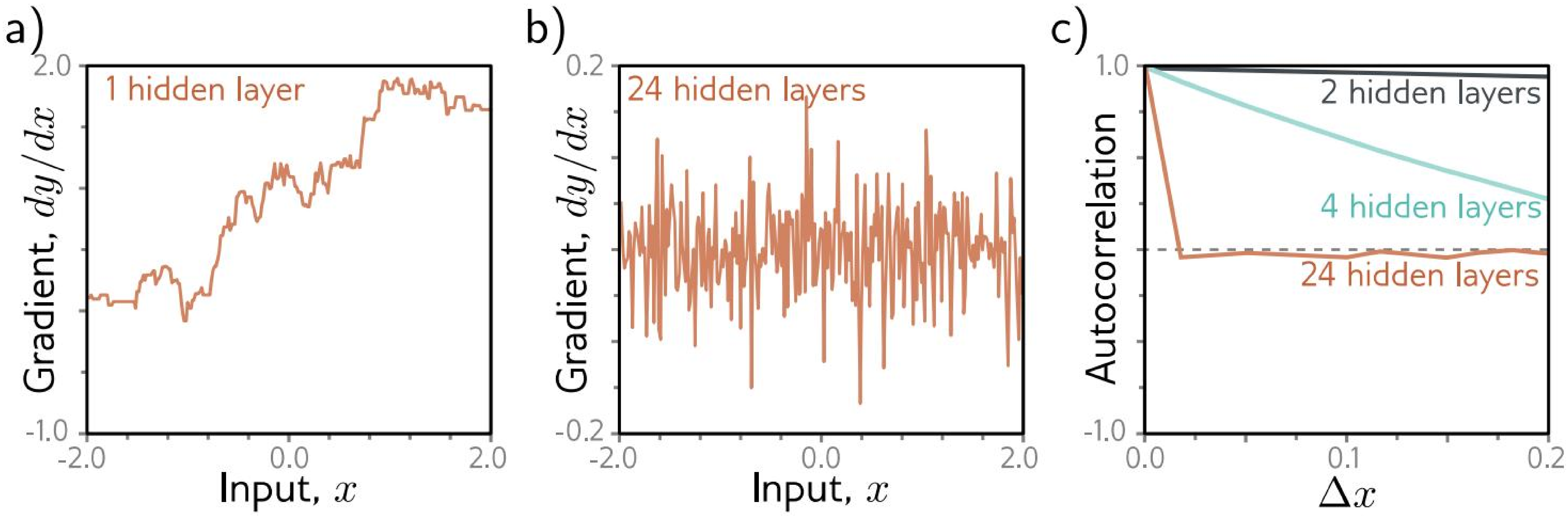
[K. He, X. Zhang, S. Ren and J. Sun, Deep Residual Learning for Image Recognition, 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2016, pp. 770-778]

Motivation

1. Gnerally, more complex models may lead to overfitting, with worse performance in test data
2. However, it may decrease the training error due to "overfitting"
3. That means, the problem is training deeper neural networks, not their generalizatoin
4. This phenomenon is not completely understood, but one conjecture is at initialization
   - Loss gradients change unpredictably when we modify parameters in early network layers
   - With reasonable initialization, we can avoid gradient exploding or vanishing
   - However, the derivative assumes infinitesimal step size,
     - In practice, our step size is finite
     - It may cause problems if the loss surfice looks like enormous range of tiny mountains rather than a smooth one
   - This conjecture is supported by empirical observations of gradients in networks with a single input and out

Motivation

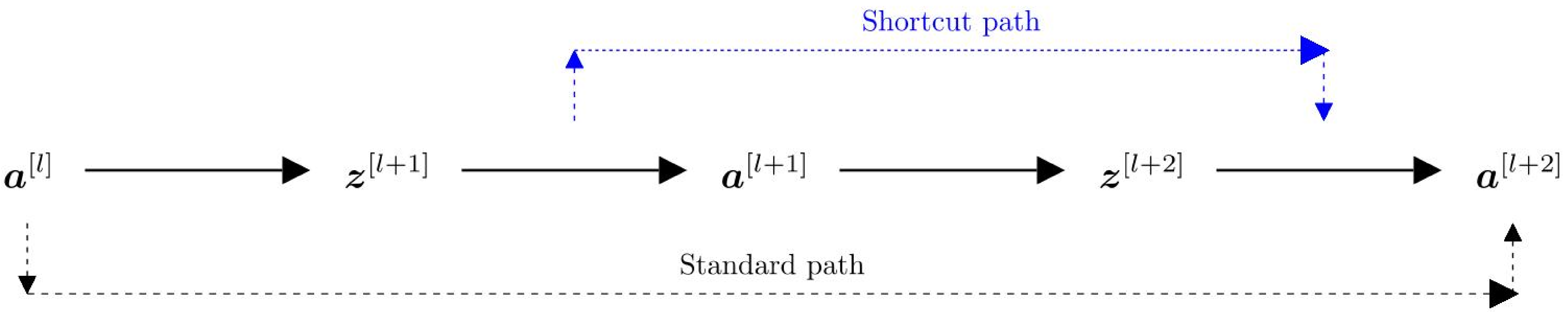
1. The following image is Figure 11.3 of Prince (2024)



Motivation

1. We conclude
   - For shallower networks, the gradient of output with respect to the input changes slowly
   - For deeper networks, however, it is not this case
2. This is captured by the autocorrelation function of the gradient
   - For shallower networks, nearby gradients are correlated
   - For deeper networks, however, it is not this case
3. This is termed the *shattered gradients* phenomenon

Structure



$\boldsymbol{z}^{[l+1]} = \boldsymbol{W}^{l+1}\boldsymbol{a}^{[l]} + \boldsymbol{b}^{[l+1]}$ $\quad$ $\boldsymbol{a}^{[l+1]} = \sigma^{[l+1]}(\boldsymbol{z}^{[l+1]})$ $\quad$ $\boldsymbol{z}^{[l+2]} = \boldsymbol{W}^{l+2}\boldsymbol{a}^{[l+1]} + \boldsymbol{b}^{[l+2]}$ $\quad$ $\boldsymbol{a}^{[l+2]} = \sigma^{[l+2]}(\boldsymbol{z}^{[l+2]})$

1. Shortcut path

$$\boldsymbol{a}^{[l+2]} = \sigma^{[l+2]}(\boldsymbol{z}^{[l+2]} + \boldsymbol{z}^{[l+1]})$$

2. Thus, $\boldsymbol{z}^{[l+2]}$ models the "residual" associated with $\boldsymbol{z}^{[l+1]}$ and $\boldsymbol{z}^{[l+2]}$

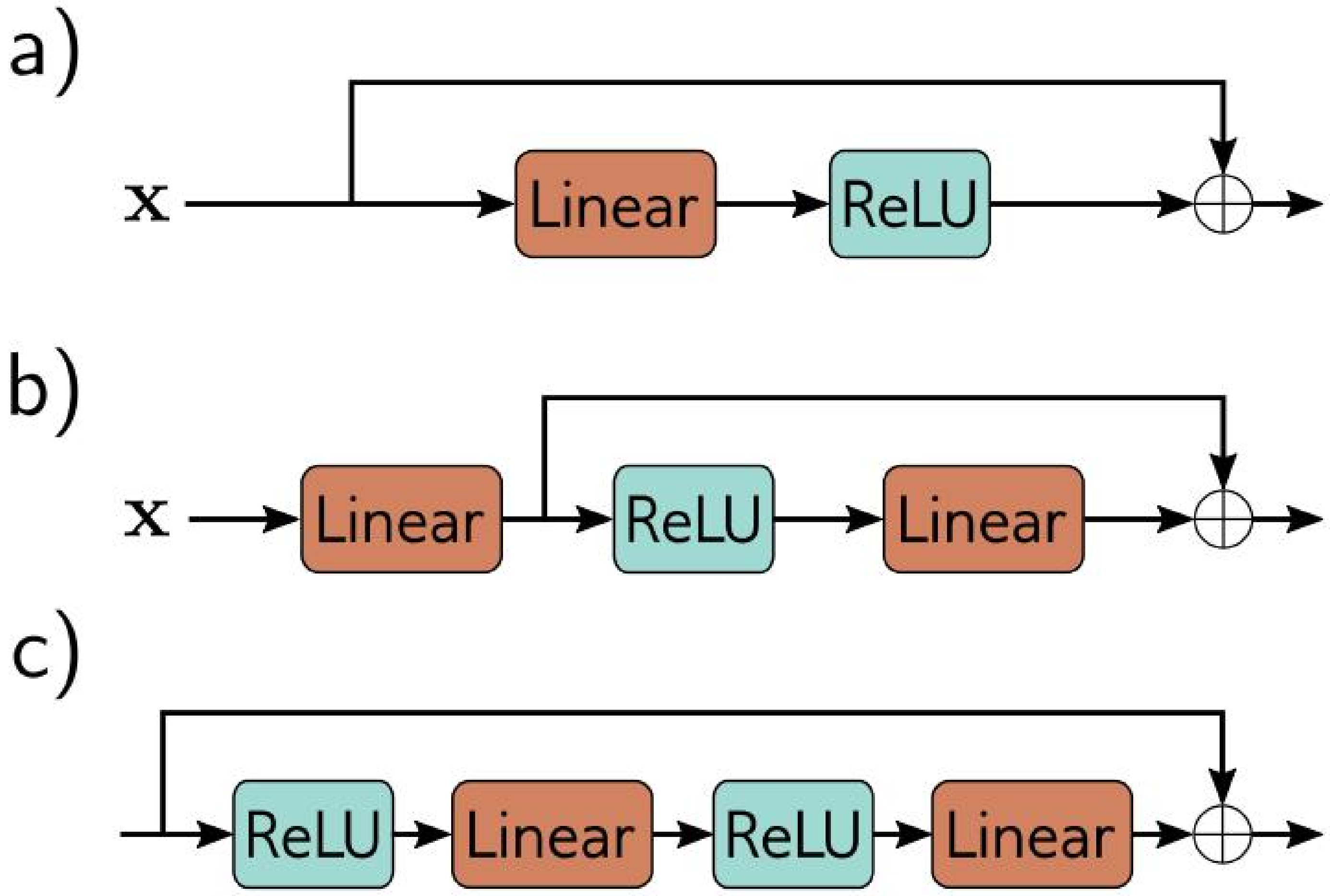
Remark

1. More general, the length of $\boldsymbol{z}^{[l+2]}$ may be different from that of $\boldsymbol{z}^{[l+1]}$
2. This problem can be easily solved by introducing a new parameter $\boldsymbol{W}_s^{[l]}$

$$\boldsymbol{a}^{[l+2]} = \sigma^{[l+2]}(\boldsymbol{z}^{[l+2]} + \boldsymbol{W}_s^{[l]} \boldsymbol{z}^{[l+1]})$$

Remarks

1. The following image if Figure 11.5 of Prince (2024)



Remarks

1. Notice that the outpur after ReLU activation is usually nonnegative
2. Thus, if we apply ReLU right after ReLU activation, we can only increase the values of "input"
3. Residual connection usually joins two linear transformation results

Remarks

1. Residual connection can be used to deepen neural networks
2. Thus, we may suffer from gradient vanishing or exploding (exponentially)
3. A typical technique to alleviate this difficulty is to use *batch normalization*